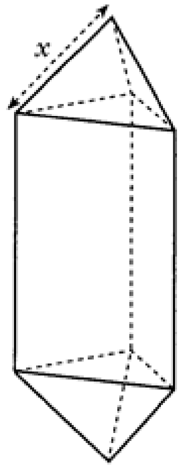
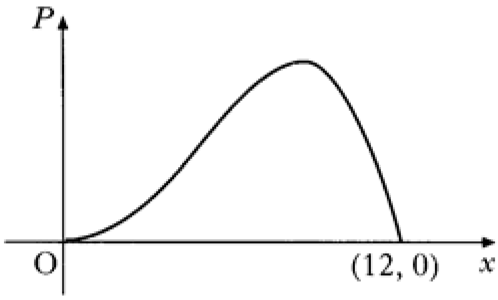
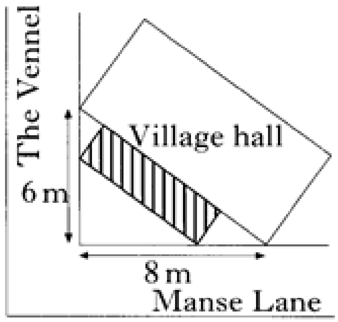
Optimisation

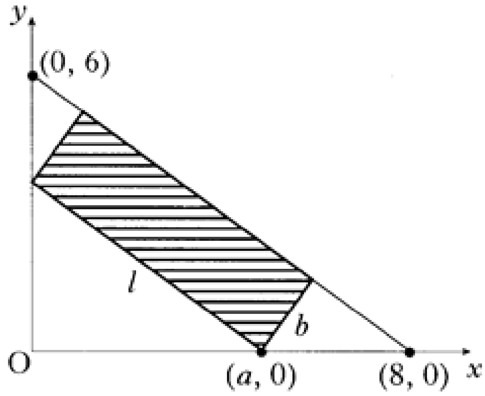
2000 P2	**A6.** A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end. The surface area, A, of the solid is given by $$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$ where x is the length of each edge of the tetrahedron. Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.  **6**
2001 P1	**6.** A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$. Find the value of x which gives the maximum profit.  **5**

2002 P2

10. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, $A\ \text{m}^2$, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

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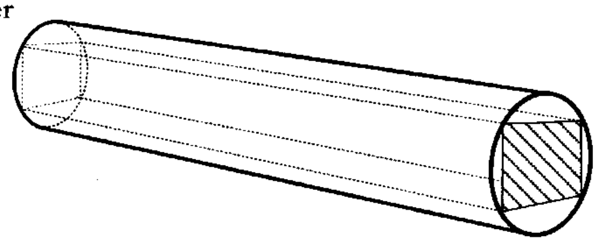
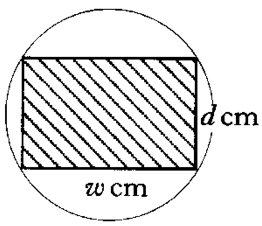
3. JAN) 02 P2

A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.



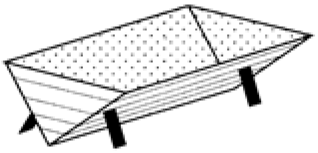
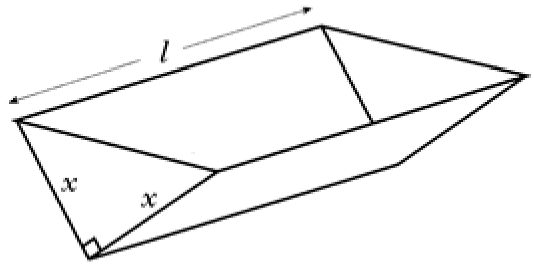
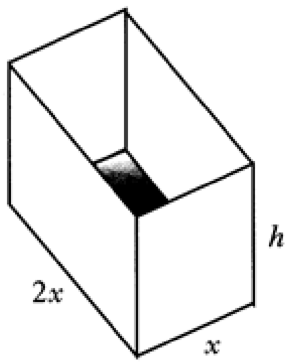
The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.



The strength S of the beam is given by

$S = 1{\cdot}7\,w\,(400 - w^2)$.

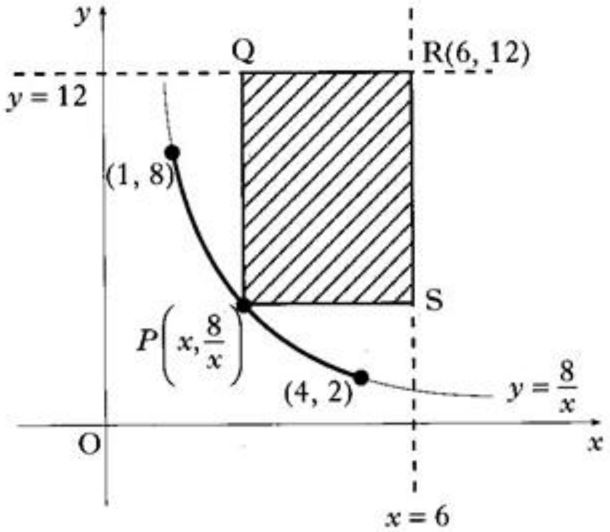
Find the dimensions of the beam for maximum strength. **5**

2003 P2	**8.** An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.  The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.  (*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3** (*b*) Find the value of x which minimises this surface area. **5**
2004 P2	*Marks* **9.** An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.  (*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. **3** (*b*) Find the exact value of x for which this volume is a maximum. **5**

2006 P2

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by

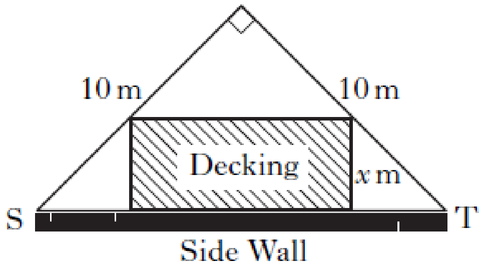
$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

2007 P2

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.

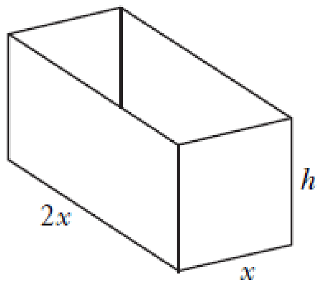
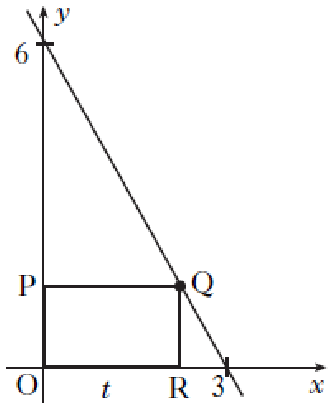
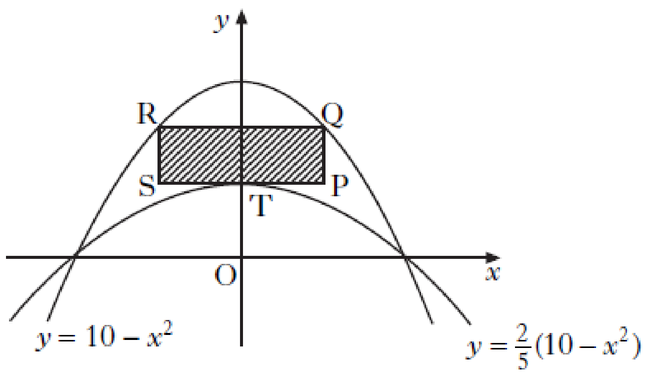


(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

(*b*) Find the dimensions of the decking which maximises its area. **5**

2008 SP2 P2	**7.** An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.  (*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. (*b*) Find the exact value of x for which this volume is a maximum.	 **3** **5**
2008 P2	**6.** In the diagram, Q lies on the line joining (0, 6) and (3, 0). OPQR is a rectangle, where P and R lie on the axes and OR = t. (*a*) Show that QR = $6 - 2t$. (*b*) Find the coordinates of Q for which the rectangle has a maximum area. 	 **3** **6**
2010 P2	**5.** The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.  A rectangle PQRS is placed between the two parabolas as shown, so that: • Q and R lie on the upper parabola; • RQ and SP are parallel to the x-axis; • T, the turning point of the lower parabola, lies on SP. (*a*) (i) If TP = x units, find an expression for the length of PQ. (ii) Hence show that the area, A, of rectangle PQRS is given by $A(x) = 12x - 2x^3$. (*b*) Find the maximum area of this rectangle.	 **3** **6**

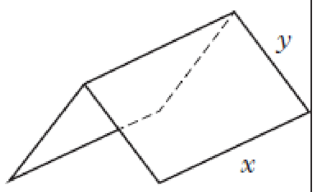
2013
P2

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

3

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

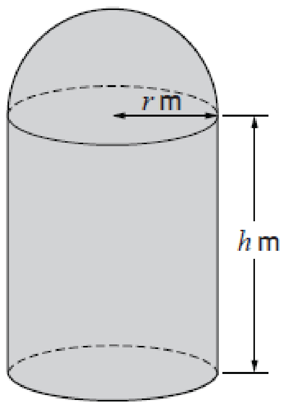
(ii) Calculate the minimum cost of a frame.

7

2015
SP
P2

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the cylindrical part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

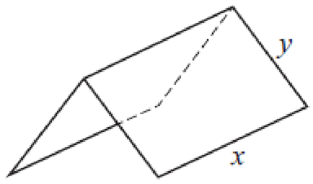
3

(b) Determine the value of r which minimises the amount of metal needed to build the container.

6

2015 EP P2

9. A manufacturer is asked to design an open-ended shelter, as shown:



The frame of the shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.

The total length, L metres, of the rods used in a shelter is given by:

$$L = 3x + \frac{48}{x}$$

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(a) Find the value of x for which L is a minimum. **5**

The rods used for the frame cost £8·25 per metre.

The manufacturer claims that the minimum cost of a frame is less than £195.

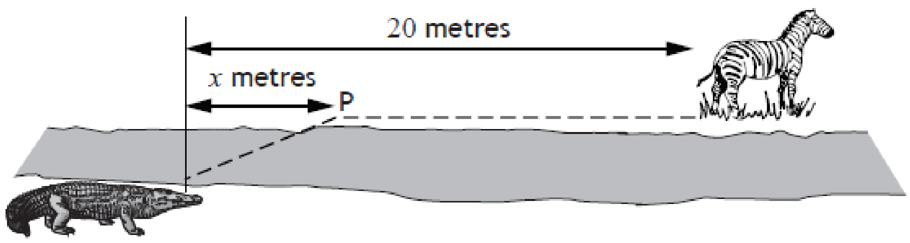
(b) Is this claim correct? Justify your answer. **2**

2015 P2

8. A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



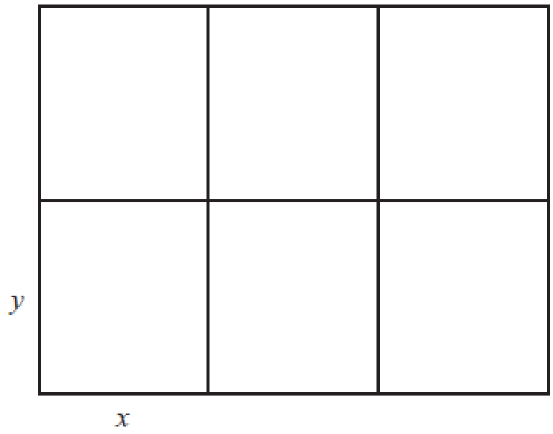
The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

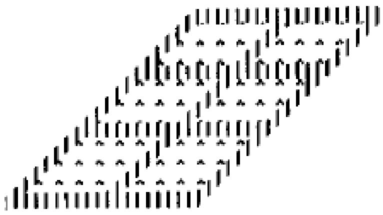
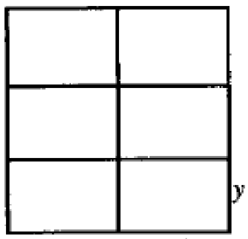
(a) (i) Calculate the time taken if the crocodile does not travel on land. **1**

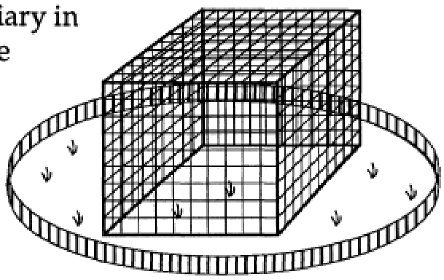
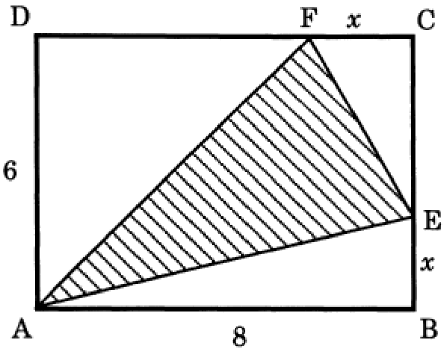
(ii) Calculate the time taken if the crocodile swims the shortest distance possible. **1**

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. **8**

2016 P2	**7.** A council is setting aside an area of land to create six fenced plots where local residents can grow their own food. Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.  (a) The area of land being set aside is $108\ \text{m}^2$. Show that the total length of fencing, L metres, is given by $$L(x) = 9x + \frac{144}{x}.$$ **3** (b) Find the value of x that minimises the length of fencing required. **6**

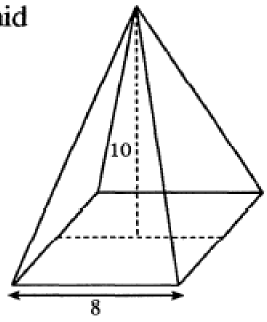
Pre 2000 - Optimisation

1	A zookeeper wants to fence off six individual animal pens.   Each pen is a rectangle measuring x metres by y metres, as shown in the diagram. (a) (i) Express the total length of fencing in terms of x and y. (ii) Given that the total length of fencing is 360m, show that the total area, $A\ \text{m}^2$, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. (b) Find the values of x and y which give the maximum area and write down this maximum area. **4, 6**

2	The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.  (*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by $A = x^2 + \frac{2000}{x}$. **(4)** (*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**
3	An yacht club is designing its new flag. The flag is to consist of a red triangle on a yellow rectangular background. In the yellow rectangle ABCD, AB measures 8 units and AD is 6 units. E and F lie on BC and CD, x units from B and C as shown in the diagram.  (*a*) Show that the area, H square units, of the red triangle AEF is given by $H(x) = 24 - 4x + \frac{1}{2}x^2$. **(4)** (*b*) Hence find the greatest and least possible values of the area of triangle AEF. **(8)**

4

A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm. and a vertical height of 10cm.

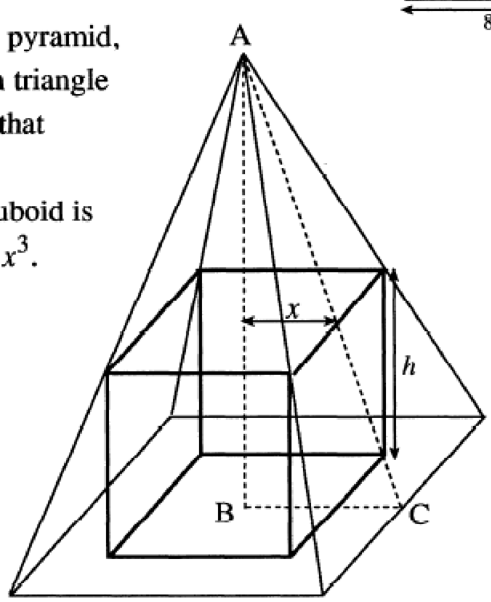


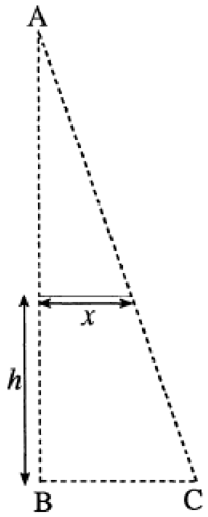
(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.

If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x$. **(3)**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**

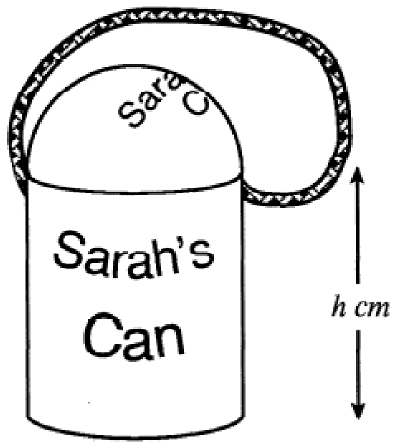




(*b*) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

5

A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \dfrac{800}{r}$. **(3)**

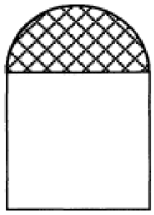
Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

6

A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.

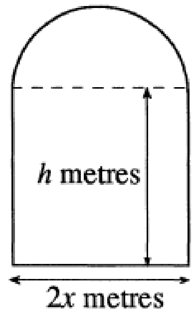
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.

(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

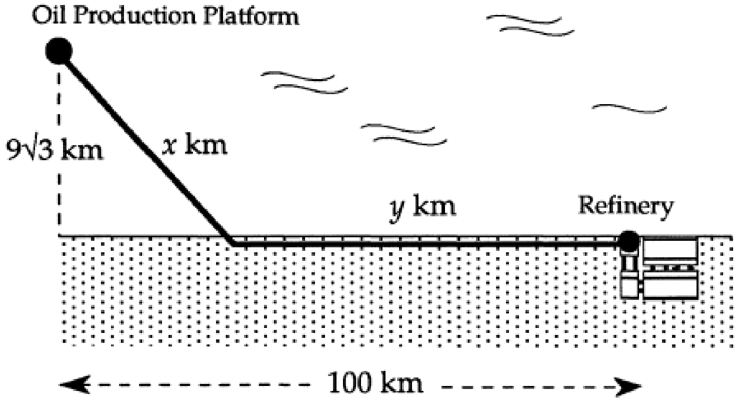
(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**



(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

7

An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

8

Diagram 1 is an artist's impression of a new warehouse based on the architect's plans. The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.
With coordinate axes as shown in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.

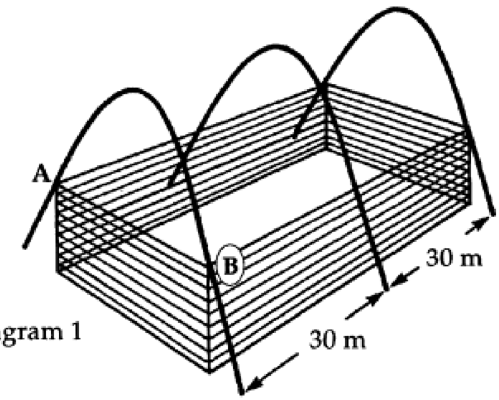


Diagram 1

(*a*) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^2\right)$ square metres. **(2)**

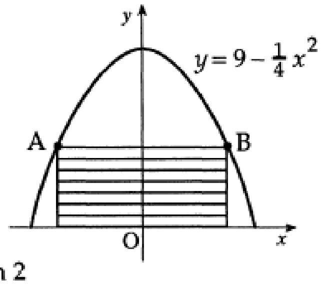
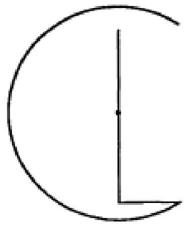


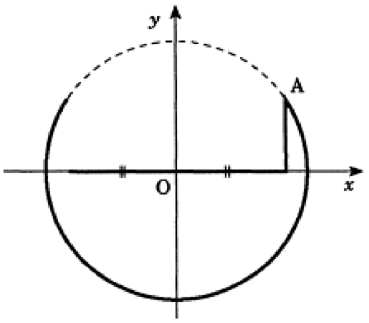
Diagram 2

(*b*) The architect wished to fit into the girders the cuboidal warehouse which had the maximum volume. Find the value of this maximum volume. **(6)**

9

Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.





The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

(a) If A has co-ordinates (x,y), show that the total length T of the arms of the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. **(1)**

(*b*) Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(*c*) By squaring both sides, solve this equation.
Hence find the greatest length of the arms of the 'L'. **(3)**